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The Relative Merits of Investable Hedge Fund Indices and of Funds of Hedge Funds in Optimal Passive Portfolios

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ABSTRACT

Can the new investable hedge fund indices (IHF) enhance the performance of optimal passive portfolios made of equities and bonds? How do they compare to funds of hedge funds (FoHF) as well as to other alternative investments such as commodities and volatility? The conclusions depend crucially on forecasts of future expected excess returns for all assets as well as a careful conditioning of the data to reflect trading costs and remove unrealistic serial correlations. A naïve forecast based on recent historical performance leads to no allocations to either IHF or FoHF, a result explained by the performance of equities and commodities and limited diversification effects from hedge funds. Yet a forecast based on market equilibrium returns for all main asset classes but hedge funds, which are kept at their historical level, leads to the opposite result with optimal portfolios almost exclusively invested in hedge funds. Both conclusions are unrealistic and unstable. More reasonable allocations are obtained with the Black-Litterman (BL) approach to combining subjective views with equilibrium returns. Then both hedge funds instruments play a significant role in optimal passive portfolios if their expected excess returns are at least 1%. Long volatility positions are also likely to be attractive. However the BL approach can also be criticised.

JEL Classification Codes: G11, C53

Keywords: hedge funds, investable hedge fund indices, funds of hedge funds, commodities, VIX, mean-variance analysis, Sharpe Ratio, Adjusted Sharpe Ratio, Omega Ratio, Black-Litterman model.

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I COMPARING INVESTABLE INDICES TO FUNDS OF HEDGE FUNDS

Investable hedge fund (IHF) indices have been introduced to provide an alternative to fund of hedge funds (FoHF) investments and facilitate access to the reputed skills of hedge fund managers. The first investable index was launched in May 2002 by Standard & Poor's (S&P), and was rapidly followed by launches from Hedge Fund Research (HFR), CSFB/Tremont (CSFBT) and others. Index sponsors support one or more composite indices and a number of hedge fund strategy indices. These indices are promoted as a transparent[1] way to invest in hedge funds with the choice of matching either the entire hedge fund universe or baskets of single strategy hedge funds. In reality, one cannot trade investable indices directly, but index sponsors (and other firms) offer, at a price, a variety of investment vehicles, such as indexed bonds and swaps, with performance linked to their investable indices. Capital flows into IHF index based investment vehicles are substantial and growing. Already in March 2004, HFR advised that investments linked to their investable indices exceeded US$2 billion. In March 2005, CSFBT reported their investable indices exceeded US$3 billion. By July 2006, investments in investable indices linked products must have exceeded US$20 billion.

Index sponsors impose strict selection criteria on their participating hedge funds to ensure liquidity, transparency and security. The funds must be highly liquid, meet minimum assets under management levels, and possess a track record of appropriate performance. The sponsors also conduct extensive quality screening, perform ongoing due-diligence and require hedge fund performance to be audited. But these selection criteria may entail hidden costs for investors in the form of adverse performance selection. Hedge funds with superior performance records may find no need to be included in investable indices as they are already able to attract and retain investors, whereas less performing hedge funds may be more willing to meet the selection criteria in the hope of attracting more funds. Selection criteria such as high liquidity may also be intrinsically detrimental to superior long-term performance.

On the other hand, FoHF managers claim they possess superior manager selection skills that justify the additional management and performance fees they charge. But their task and performance are evolving as interest in hedge funds continues to grow and some traditional funds are allowed to adopt some of the more typical hedge fund strategies (e.g., leveraging, shorting, using derivatives). With current estimates of more than 8000 hedge funds and $1,200 billion of assets under management, there is both increasing competition among hedge funds and crowding in some of the narrower hedge fund strategies (e.g., event driven) as well as possibly greater alignment with traditional asset classes and more 'normal' return distributions. Previous findings that hedge funds provide superior if somewhat 'non-normal' returns and attractive diversification away from traditional asset returns need therefore be re-examined.

There has been extensive research (for example Amenc & Martellini, [2002], Gueyie, [2006]) and discussions about the role hedge funds could play in traditional, largely passive, bond-equity portfolios without unequivocal conclusions. Indeed, several major institutional investors, including pension funds, have recently allocated resources to hedge funds whereas others have rejected the idea in principle.[2] We broaden the debate by investigating specifically the relative merits of FoHF and IHF in the wider context of a portfolio including not only equities and bonds but also two other alternative investments: commodities and volatility[3]. Like hedge funds, commodities and volatility are known to provide diversification away from equities and bonds; they might be more attractive than hedge funds in optimal portfolios. We carry out our comparisons over a broad range of return forecasts including the Black and Litterman (BL) [1992] approach and in the context of several optimization criteria to seek robust results. We reach some general conclusions but find that all forecasting methods we used, even the BL approach have their flaws.

In Section II we review previous research on alternative investments. In Sections III and IV we explain our selection of investment proxies for the main asset classes and the preparation of the return data. The choice of forecast scenarios, risk adjusted performance measures and the optimization process are explained in Section V. The optimal allocations are discussed in Section VI and we conclude in Section VII.

II PREVIOUS RESEARCH AND MOTIVATION FOR ALTERNATIVE INVESTMENTS

Interest in alternative investments keeps growing. Previous research has highlighted the relative advantages and peculiarities of some of these investments: with hedge funds, low volatility and low correlation with all other asset classes as well as good, if somewhat 'non-normal' returns; for commodities, also low correlation and potentially high returns but high volatility; for volatility, poor returns, high volatility and limited liquidity but significantly negative correlation with equities.

II.1 Hedge Funds

Fung and Hsieh [1997] identify that performance improves substantially when hedge funds are included in a traditional portfolio. However, Amin and Kat [2003] show that the inclusion of hedge funds comes at the cost of lower skewness and higher kurtosis. Ranaldo and Favre [2004] also conclude that the lack of consideration of the higher moments leads to insufficient compensation for the investment risk of hedge funds. Due to these deviations from normality, integrating hedge funds into a traditional portfolio is a complex matter [Kat, 2005]. Kat demonstrates that the undesirable skewness and kurtosis in hedge funds can be neutralised by investing in out-of-the-money equity put options. But, recent research [Black, 2006] indicates that investing in out-of-the-money put options may be an expensive way to reduce unwanted skewness and kurtosis. Black shows that a small investment in VIX futures can, also, neutralize these undesirable properties.

II.2 Commodities

The literature identifies diversification benefits from investments in commodities. Greer [1978] shows that adding a commodity futures index to a portfolio of large capitalisation stocks improves the risk return profile. Bodie [1983] shows a similar effect when adding commodity futures to a portfolio of equities, bonds, and Treasury bills. Many other studies also assert that a passive investment in commodity futures provides diversification benefits for traditional portfolios (see Anson [1998], Gibson [1999] and Ankrim & Hensel [1993]).

The growing interest from investors in commodities[4] is also driven by recent macroeconomic factors. Akey [2005] argues that increases in global consumption, low inventory stockpiles, expectations of increased inflation, a weak US dollar, a rebound from weak historical prices and supply limitations favour long term performance of investments in commodities. As long as these factors persist, commodities should increasingly attract investors. Commodity futures and options provide easy access to this sector.

II.3 Volatility

It is well known that in equity markets volatility movements are negatively correlated to prices. For example, Black [2006] observes that the S&P-500 volatility index VIX rises quickly during market crises; Alexander [2004] analyses the negative correlation between volatility and index variations for the FTSE-100. Long volatility positions should therefore provide high diversification benefits to traditional equity investors. Volatility trading is still embryonic, but its potential advantsges to investors are being recognized and trading volumes are now increasing rapidly.

A pure volatility trade should provide exposure to volatility alone without being affected by the directional movements of the underlying asset [Hafner & Wallmeier, 2006]. Common methods for trading volatility include buying and selling option ‘straddles’ or ‘strangles’ and then delta hedging with the underlying security. However, these strategies require frequent rebalancing and incur substantial trading costs. Better alternatives are now available. There are listed futures and options contracts based on the S&P 500 VIX index as well as a few other equity indices. Liquidity in these contracts has recently picked up. There is also a growing OTC market in variance swaps.

III SELECTION OF INVESTMENT PROXIES AND DATA SOURCES

Our investment universe consists of six proxies. For the IHF proxy we select the original S&P composite index for which we have the longest series of monthly net asset values going back to September 2002. We create a FoHF proxy from funds in the Eurekahedge FoHF database. Commodities are represented by the Reuters-CRB commodities index and volatility by the Chicago Board Options Exchange volatility index VIX. The traditional asset classes are represented by the total return S&P 500 index and the US$ hedged total return Lehman Brothers Global Aggregate bond index. The full data set for the six proxies consists of 45 monthly observations from September 2002 until June 2006. This period covers the end of the 'tech-wreck' and the aftermaths of the 11 September 2001 terrorist attack and the Iraq War. The period includes a strong recovery in equity markets since March 2003, a rapid economic growth in the BRIC nations[5] and a secular 'bull market' in commodities. During this period, the Federal Funds rate dipped from 1.75% in September 2002 to 0.75% in September 2003 to reach 5.25% in July 2006;[6] correspondingly, the bond markets have weakened. Because a prime aim of this research is to compare the relative merits of IHF and FoHF, we use only these data for estimating correlations between returns. However, in our equilibrium scenarios, we also use monthly observations over ten years to obtain more stable volatility estimates for the other asset classes. We explain below the reasons for our choice of proxies. We shall refer to them simply as Equity, Bond, Volatility, Commodity, IHF and FoHF.

III.1 Equity

We select the US dollar total return S&P 500 index as a proxy for equity markets. This choice introduces a US bias towards equities. An alternative would have been to choose an international equity index such as one of the MSCI Equity World indices. However the S&P500 represents a large fraction of world equities and now contains a large number of international companies. Major equity markets outside of the US are also highly correlated with the S&P 500 index. We verified that the broader MSCI Equity World Index with returns hedged into US dollars would only marginally increase the attractiveness of the equity asset class compared to using the S&P 500 over the period of interest. In the end, we choose the S&P500 over the MSCI Equity World Index because we are also using a volatility index based on the S&P500 as an alternative investment proxy.

III.2 Bond

We adopt the Lehman Brothers Global Aggregate bond index as the proxy for bonds. This index is representative of global markets for government bonds, corporate credit and mortgage-backed securities. We use the US dollar fully hedged series. Both our equity and bond proxies are expressed in US dollars like all our other proxies.

One will note that our choice of the S&P 500 index and of a fully foreign currency hedge bond index does not leave any room for potentially attractive foreign exchange exposures. It has been argued that a small degree of foreign exchange exposure is desirable in an optimal international portfolio [Black, 1989][7]; however, the optimal exposure is likely to be of the order of 20% of foreign investments only, therefore full hedging of foreign currency exposure ought to be closer to the optimum balance than no hedging at all. We verified that taking a small degree of foreign exchange exposure would not affect significantly the relative merits of the asset classes we consider.[8]

III.3 Volatility

We adopt the Chicago Board Options Exchange (CBOE) volatility index VIX as the proxy for a passive investment in volatility. The VIX is a measure of the 30-day forward implied volatility for the S&P 500 index. (SPX)[9] The VIX squared is the reference for the 30-calendar-day SPX variance swap. The CBOE Futures Exchange (CFE) listed VIX futures in March 2003 and VIX options in February 2006. Trading volumes were slow to pick up but have dramatically increased recently.[10] There are now also futures traded on other equity volatility indices (Vstoxx, Vdax and Vsmi). We choose the spot index rather than any futures price so that we are not influenced by the term structure of the futures contracts.

III.4 Commodity

We use the Reuters-CRB commodity index because it is equally weighted in the main commodity sectors[11] and we did not want the index to be dominated by a particular sub-sector. It consists of 17 physical commodities. No consideration is given to liquidity or production volumes in the weightings for each commodity. As for volatility, we use the spot index as opposed to any futures prices.

III.5 Investable Hedge Fund

Since IHF indices are relatively new, we briefly review their development. The exact selection criteria and due diligence processes adopted by the sponsors of investable indices have been presented in previous research [Gehin & Vaissie, 2004]. The largest sponsors of IHF are S&P, CSFB/Tremont, MSCI, Dow Jones and HFR.

For example, S&P, the leading rating agency and index provider, first introduced four investable indices on 30 September 2002, a Managed Futures index on 31 December 2002 and another three indices on 31 March 2004. The original S&P composite index was launched with forty constituent hedge funds. PlusFunds Group Inc. acts as the investment manager and offers investment products based on the

S&P investable indices. The funds are monitored for style drift and performance. At launch the indices were equally weighted. Since then they are rebalanced each year on January 1st each year at the discretion of the investment manager.

A possible choice of IHF proxy for our comparison could be one of the largest composite investable indices, provided its performance is representative of the sector. We select the S&P composite investable index for two reasons: it has the longest history (45 months of data July 2006) and we verified that its performance statistics are well within the range of its competitors, CSFBT, MSCI and HFR. A weighted average of major investable indices would have a shorter history and would probably exhibit less volatility because of the added degree of diversification.

III.6 FoHF

The Eurekahedge FoHF database contains time series of monthly returns net of management and incentive fees in US dollars. We do not want to use a basket of FoHFs as a proxy as it would not be representative of the volatility of a single FoHF. However, it would be too arbitrary to choose a single FoHF as representative of the sector. We resolve this issue as best we can by constructing an average of 45 funds chosen at random: 30 that reported continuously during the 45 month-period to July 2006, 8 that started reporting after the commencement of the period and 7 that stopped reporting during the period. The missing data for these funds is randomly filled from the data of other FoHF without repeated drawings from any individual fund. The returns for this basket are then adjusted to match the average of the volatilities of the funds in the basket.

IV DATA CONDITIONING AND PERFORMANCE OF PORTFOLIO CONSTITUENTS

Comparisons among our six proxies would not be fair unless we took into account typical costs for trading these instruments. Indeed, the source data for the IHF indices do not reflect the costs of trading investment products based on these indices. Costs often include one-off subscription and redemption fees as well as yearly management fees. These costs are particularly significant for IHF investment vehicles with daily liquidity[12]. Many funds of hedge funds also charge redemption fees not reported in their performance data. To reflect these costs, both annual and singular, on an annualized basis, we must refer to some investment planning horizon. We assume that two years is a sensible horizon for investors considering hedge funds. On this basis the average annualised trading cost for an IHF product is about 2% and for a FoHF about 0.5%. We therefore reduce the annual returns of our IHF and FoHF proxies by these figures. Note that a longer investment planning horizon would be more favourable to IHF rather than IHF and, vice versa, because FoHFs have relatively lower redemption fees than the fixed fees of IHF based products. Annualised trading costs for all other asset classes are considered negligible over the same two-year investment planning horizon. Finally, for all six proxies we calculate excess returns by subtracting the relevant US Treasury Bill rates. Henceforth, we use the term 'return' for short instead of 'excess return'.

Previous research [Brooks & Kat, 2002] identifies the presence of positive autocorrelation in many hedge fund return series, as if lack of liquidity induces some fund managers to smooth their valuations. Positive autocorrelation, if not adequately taken into account, would lead to an underestimation of long-term volatility and therefore an overestimation of risk adjusted performance measures. Any autocorrelation significantly different from zero over a time lag typical of a trading interval is unlikely to persist in a liquid, efficient market; it would lead to obviously profitable trading strategies. Our autocorrelation estimates for the six investment proxies are shown in Exhibit 1; they indicate positive one-month autocorrelations of 0.37 and 0.33 for the IHF and FoHF proxies. On the other hand, the volatility index has a negative autocorrelation of –0.32 reflecting both the strong mean reverting properties of this index and lack of trading, or very low trading volumes, until recently. The autocorrelation figures for the other assets are small and not very significant, as we would expect with liquid markets[13]. We choose therefore to remove the serial correlation from the two hedge funds and the volatility proxies, but not to adjust the returns of the other proxies.

We remove serial correlation by applying the simple Blundell-Ward filter.[14] If one assumes that the reported returns r_t follow the AR(1) process

$$r_t = \alpha + \rho r_{t-1} + \varepsilon_t \tag{1}$$

where α and ρ are constants and the ε_t are i.i.d. random variables, then the series r_t^* defined by

$$r_t^* = \left(r_t - \rho r_{t-1}\right) / \left(1 - \rho\right) \tag{2}$$

should have zero autocorrelation and essentially the same mean as r_t. The autocorrelation ρ is derived from the AR (1) regression of the original r_t series.

The Table in Exhibit 1 displays the key statistics for the adjusted returns expressed on an annualised basis. One observes the returns are dissimilar except perhaps for two pairs: Bond and IHF on the one hand, Equity and Commodity on the other. Volatility clearly stands out with both negative return and large volatility (Sharpe ratio of –0.53). This would make long volatility positions extremely unattractive if it were not for their negative correlation of with equity. These results are also shown graphically below the Table.

The higher moments of each of these series show only minor deviations from normality. Again the only proxy that stands out is Volatility with some positive skewness and excess kurtosis. These statistics may surprise; they seem to contradict previous analyses showing negative skewness and positive excess kurtosis in hedge fund returns. Instead, we observe very small but positive skewness and negative excess kurtosis. There may be two or three explanations. The first is that we consider broadly based hedge fund proxies: the S&P investable composite index has around 40 constituents; a FoHF is invested typically in 20 to 40 hedge funds. Diversification not only reduces volatility, but is also likely to reduce skewness and kurtosis. The second explanation is purely statistical: many studies have analysed skewness and kurtosis of monthly returns. If successive monthly returns are largely independent, monthly skewness and kurtosis translate into much lower figures for annual returns. The third possible explanation is conjectural: as the hedge fund industry matures and given the strong performance of equities and commodities over the study period, many hedge fund managers have been attracted to taking equity and commodity exposures. At any rate, non-normal returns do not seem to be an issue with any of our investment proxies. This point is confirmed later when we find almost identical optimal allocations for a range of risk adjusted performance measures.

The correlation matrix in Exhibit 2 displays two noticeable features. First, we have confirmation that Volatility is negatively correlated with Equity (-0.5), IHF (–0.41) and FoHF (–0.42) but hardly correlated with Bond and Commodity. Second, we observe that over the last few years, hedge funds proxies have been positively correlated with Equity and Commodity (correlations between +0.45 and +0.53) and slightly with bonds (+0.20 and +0.23). There was little correlation between Equity, Bond and Commodity. The diversifying role of hedge funds in equity portfolios has waned over the last 3 years.

V OPTIMIZATION FRAMEWORK

Expressing plausible views about future returns is the key challenge in analysing the effects of adding new asset classes to a traditional portfolio. With our proxies, the issue of non-normality of returns and therefore choice of an appropriate risk adjusted performance criterion will be seen as secondary. It is well known that traditional Markovitz type mean-variance portfolio analysis produces optimal allocations that are very sensitive to views on future expected returns, views which, even with the help of expert analysts, remain highly uncertain. This has been hailed as the main reason why professional portfolio managers do not make extensive use of traditional mean-variance portfolio analyses, and, when they do, they use the results only indications of the directions in which they might want to tilt their portfolios.

We consider three types of forecasts, or scenarios: a naïve 'Historical' scenario, a market 'Equilibrium' scenario for all asset classes but hedge funds for which we explore a range of possibilities, and, finally a 'Subjective' scenario in which we assign a degree of uncertainty to our forecasts of expected returns of the hedge fund proxies versus equilibrium expected returns for other asset classes.

IV.1 The Three Return Scenarios

The most naïve way to forecast future returns is to make the assumption of continuity, that is, to assume that statistical properties of returns observed in the recent past will persist. We call this the 'Historical' scenario. A long history of returns would reduce statistical estimation errors if the return processes had stable parameters, but rare are the portfolio managers who believe that the distant past is of much relevance to explain future returns. We therefore use our Historical scenario over the 45 months to July 2006 as an illustration only, and not necessarily as a good starting point for our investigations. The expected returns, the volatilities and the correlation matrix for this 'Historical' scenario are shown in Exhibits 2 and 4(i).

Black and Litterman [1992] argued that it would be more reasonable to start from a neutral set of expected return forecasts than from recent historical averages. By neutral set they mean a set of expected returns that would justify the optimality of currently observed global asset allocations (or the current allocations of a particular benchmark fund if the fund manager wants to define an active portfolio relative to a benchmark). In the Markovitz mean-variance analysis context the column vector of 'equilibrium' expected returns, $\boldsymbol{\pi}$, is related to optimal allocations and covariances as follows:

$$\boldsymbol{\pi} = \gamma\boldsymbol{\Sigma}\mathbf{w} \tag{3}$$

where $\mathbf{w}$ is the column vector of current market allocations, $\boldsymbol{\Sigma}$ is the covariance matrix of returns and the scalar γ is a risk aversion coefficient.[15] Note that the equilibrium expected returns $\boldsymbol{\pi}$ vary proportionally to the subjective risk aversion coefficient γ. Some indication of the 'market' risk aversion coefficient can be obtained by matching historical (or consensus) returns with $\boldsymbol{\pi}$. Risk aversion coefficients from 2.5 to 5 have been used in the literature; we choose 4 arbitrarily but in good company.[16] Our results are not very sensitive to this choice.

Thus we generate equilibrium returns for all asset classes except for the two hedge fund proxies. The equilibrium market weights are negligible for all but Equity and Bond; they are shown in Exhibit 3. But since we longer time series of historical data available for the non-hedge fund proxies and we know that there is some stability in long-term volatility estimates, we use 10 years of monthly data to estimate the volatilities of non-hedge funds proxies. For the two hedge funds proxies we explore a range of possible returns from their 45 months historical mean returns (base case) down to zero expected returns, which is about their equilibrium return. The annual expected returns and volatilities for the base case 'Equilibrium' scenario are shown in Exhibit 4(ii).

In the next Section we find that optimal allocations are very sensitive to the HF expected returns when we keep volatilities at their historical estimates. Black and Litterman (BL) argue that we should consider neither the equilibrium returns nor the subjective forecasts as certain; rather we should attribute uncertainties to each and combine them in a way that recognises these uncertainties as well as dependencies among returns. Thus, for example, since hedge fund returns are positively correlated with equity returns, an increase in hedge fund expected returns should cause an increase in equity expected returns. We use the (BL) method for combining uncertain forecasts of expected returns with market equilibrium information to re-cast some of our previous forecasts.

The BL approach has been described extensively in Black and Litterman (1992) Litterman and He (1999) and Drobetz (2001) among others. Two types of parameters are needed to describe the degrees of uncertainty in the market equilibrium expected returns, on the one hand, and in subjective forecasts, on the other. One should reflect on the meaning of these assignments. Firstly, why should equilibrium expected returns be considered as uncertain? Indeed, the estimation of the variance-covariance matrix of returns based on monthly observations already takes into account the sample standard error in the monthly mean returns. Of course, the standard error for the annualised means is about 12 times larger. As a proportion of the yearly volatility estimator, the uncertainty in the annualised mean (based on 45 monthly observations in our case) is therefore about equal to $(12/45)^{1/2} = 0.516$. But the BL approach relies on the equilibrium returns derived from (3) and therefore the sources of uncertainty are (i) statistical errors in the evaluation of the covariances matrix, (ii) uncertainty in assessing the market risk aversion coefficient γ, and (iii) model error (the price dynamics we assume – in particular, constant volatility – are a simplifications of reality and the parameters we estimate may not be stable). Of these three uncertainties the second about the risk aversion coefficient is easily the largest but also the one with least impact since the risk aversion coefficient acts purely as a scaling factor and does not affect market portfolio weights. BL also assume, for simplicity but plausibly, that uncertainties about equilibrium expected returns have a covariance structure proportional to $\mathbf{\Sigma}$, the covariance matrix of return. They add that "Because the uncertainty in the mean is much smaller than uncertainty in the returns itself, τ will be close to zero"[17]. In the end, we understand that uncertainty in the mean will be combined with volatility (i.e., uncertainty conditional on the mean) to produce the market forecast distribution of returns so, given the statistical error in the estimator of historical mean we mentioned earlier, we choose $\tau = 0.16$, that is a standard deviations of expected returns equal to 40% of the volatilities of returns. For example, for equities the uncertainty about the equilibrium expected return is 40%(15.59%) = 6.24% but for bonds it is only 40%(2.80%) = 1.12%.

The BL approach also uses subjective and uncertain forecasts of expected returns (or combinations thereof). These uncertainties are assumed to be independent and normally distributed; they are fully specified by standard deviations. To assess these standard deviations one needs to understand how they are used. BL treat the market equilibrium information and the subjective forecasts as two independent and complementary views about expected returns. It follows that the resulting probability density for the expected returns is defined as the product (normalised) of the probability densities from each source of information. The combined multivariate normal density is centered between the expectations from the two sources and can only be equally or more precise than either one of them (i.e., each term of the resulting covariance matrix can only be less or equal to the corresponding terms in the two source covariance matrices). This approach seems artificial for two reasons. First, experts providing views are usually analysts who pore over market information; they can hardly be expected to provide independent information; the Bayesian method of updating a prior with new information (whether it is the market view that is taken as the prior and the subjective view as the new information, or vice versa) does not apply. It implies that the more experts we have available, no matter how much they may disagree, the more precise our forecast of expected returns would be! Second, most analysts, no matter how expert they are, do not claim clairvoyance and should, in principle, be willing to express their beliefs about future returns in terms of probabilities. But what sense does it make to express uncertainties about expected returns and then to combine these views with volatilities around the expected returns? It is proper with a measuring device to distinguish precision – the dispersion of successive readings – from accuracy – the error in the mean that may result from poor calibration. But when expressing a view, there is nothing like successive readings, there should be only one state of mind (barring schizophrenia) and a single probability distribution about whatever uncertain quantity one describes. It is probably because of this difficulty in interpreting the required inputs that the BL approach is not more commonly used.

In our analysis, we will explore forecasts of expected returns ranging from 0% to 5% for the FoHF proxy and 0% to 3% for the IHF proxy and we choose to attribute equal weights to each of these forecasts and to the equilibrium expected returns. We achieve this by setting standard errors of 40% of the corresponding volatilities. For example, to each forecast of expected return of the IHF proxy, we associate a standard deviation of 40%(3.73%) = 1.49% and to each forecast of expected return of the FoHF proxy, we associate a standard deviation of 40%(6.76%) = 2.70%

With this choice of parameters, we combine the equilibrium returns $\boldsymbol{\pi}$ with the subjective views and calculate the resulting distribution for the expected returns. If we call $\boldsymbol{\mu}$ the column vector of expected returns and represent the subjective views by the set of linear equations

$$\mathbf{P}\boldsymbol{\mu} = \mathbf{Q} + \boldsymbol{\varepsilon} \tag{4}$$

where $\boldsymbol{\varepsilon} \sim N(0, \boldsymbol{\Omega})$ and $\boldsymbol{\Omega}$ is the diagonal variance matrix of the error terms $\boldsymbol{\varepsilon}$, then the expected returns follow a multivariate normal distribution $N(\mathbf{m}, \mathbf{M})$ with:

$$\mathbf{m} = [(\tau\boldsymbol{\Sigma})^{-1} + \mathbf{P}'\boldsymbol{\Omega}^{-1}\mathbf{P}]^{-1}[(\tau\boldsymbol{\Sigma})^{-1}\boldsymbol{\pi} + \mathbf{P}'\boldsymbol{\Omega}^{-1}\mathbf{Q}] \tag{5}$$

$$\mathbf{M} = (\tau\boldsymbol{\Sigma})^{-1} + \mathbf{P}'\boldsymbol{\Omega}^{-1}\mathbf{P} \tag{6}$$

The return themselves follow the distribution $N(\mathbf{m}, \mathbf{M} + \boldsymbol{\Sigma})$. The subjective scenarios we explore are reported in Exhibit 4(iii). One can see in this Exhibit how changes in views about the expected returns of the two hedge fund proxies affect the expected returns for the other proxies in a way consistent with correlations. One can also appreciate that the uncertainty about the means increases slightly the volatilities of the returns compared to Exhibit 4(ii).

IV.2 Risk Adjusted performance Measures and Optimization Methodology

We found (Exhibit 1) that the return distributions for our six asset classes deviate only slightly from normality. Various degrees of skewness and excess kurtosis make traditional mean-variance measures of performance inadequate. Dybvig and Ross [1985b] find mean-variance analysis does not properly assess positively skewed returns preferred by investors. Leland [1999] also shows that mean-variance based measures of performance are inadequate noting that strategies with positive skewness will be incorrectly underrated. Research related to hedge funds emphasises the need to consider the entire distribution; see Kat [2002], [2003], [2005], [Brooks & Kat, 2002] and Agarwal and Naik [2004]. To address this issue a variety of alternative risk adjusted performance measures has been developed.

One popular measure among hedge fund managers is the Omega ratio proposed by Shadwick and Keating [2002]. For a given threshold, Omega is defined as the ratio of the expected return in excess of the threshold divided by the expected return short of the threshold, in other words, the ratio of the call option price over the put option price struck at the threshold. Omega is a monotonically decreasing function of the threshold in the same way as the cumulative distribution of return is a monotonically increasing function of the threshold; there is a one to one correspondence between the two functions and not more information in one than in the other, but some investors and fund managers find it easier to interpret Omega than the cumulative return distribution. Kazemi, Schneeweis and Gupta [2003] define (Omega – 1) as the Sharpe-Omega measure because it is closely related to the Sharpe ratio (it is indeed equal to the expected return in excess of the threshold divided by the value of the put option at the threshold) and therefore perhaps still easier to interpret. There are several other modified Sharpe ratios. An interesting one is the Adjusted Sharpe Ratio (ASR) that incorporates a penalty factor for negative skewness and excess kurtosis. The ASR can be derived from a Taylor series expansion of expected utility with an exponential utility function. When keeping to the first four terms of the expansion the ASR [Pézier, 2004] is stated as follows:

$$ASR = SR[1+\left(\mu_3/6\right)SR-\left(\left[\mu_4-3\right]/24\right)SR^2)] \tag{7}$$

where μ_3 and μ_4 are the skewness and kurtosis of the returns distribution and SR denotes the Sharpe Ratio. For the purpose of this research we maximise the Sharpe ratio, the ASR and the Omega ratio for three threshold levels above the risk free rate: 0%, +3% and +6%; this covers the typical range of returns an investor might want to aim for depending on her degree of risk aversion.

Out of the 45 observed monthly returns adjusted to match the expected returns and volatilities of the chosen scenario, we simulate 2000 yearly returns by choosing random combinations of 12 monthly returns. We then apply an optimiser that searches among the possible allocations those that maximise the chosen risk adjusted performance criterion. The search procedures start from a broad grid of potential solutions and then uses a proprietary set of genetic algorithms [Palisade, 1998] to identify the optimal allocation. For all portfolios and asset classes we assume the portfolios are fully invested. We set minimum allocation constraints of –10% and maximum of 100%, except for the hedge fund indices which cannot be shorted and where the minimum is set at zero.[18] We also run the some optimizations under the assumption of no short sales.

We are conscious of the cynical view that some optimizations are best described as 'Error Maximizers' that misallocate the weights of the portfolio components. But Kritzman [2006] identifies small errors in the estimation of the expected returns of a small number of relatively dissimilar assets, as is the case in hand, cause only in minor misallocations.

VI RESULTS

Optimal allocations are reported in Exhibits 5 to 9. We report first the results of the 'Historical' scenario in Exhibit 5. Optimal allocations under each of five risk adjusted performance measures (RAPM) are found in the top panel. Two features spring to attention. First, whatever the RAPM, there is no place for the hedge fund proxies in optimal portfolios. Second, the allocations obtained for the simple Sharpe Ratio, the Adjusted Sharpe Ratio and the Omega(0%) ratio are very similar to each other, probably within the range of accuracy of the optimizer. Main allocations go to Equity, Bond and Commodity (around 36%, 45% and 16%, respectively the residual 3% going to Volatility. Compared to market equilibrium, there is essentially a tilt from Bond to Commodity. The tilt becomes more pronounced for adventurous investors maximising Omega at 3% and 6%. When maximising the Omega ratio at +6%, Bond and Volatility reach their maximum short positions of –10% allocation; positive allocations go exclusively to Equity and Commodity. This should not surprise, considering the relatively poor performance of Bond over the period but the strong performance of Commodity. Of course these trends may have already started to reverse and probably few fund managers would trust the 'Historical' scenario.

The two main lessons from the 'Historical' scenario are therefore that (i) it leads to extreme allocations in which investors have probably little confidence and (ii) deviations from normality in return distributions do not affect optimal allocations significantly. The middle panel in Exhibit 5 displays statistics that confirm the relatively small skewness and excess kurtosis of optimal portfolios. The bottom panel in Exhibit 5 indicates that with no short sales, the no-longer authorised short positions in Bond and Volatility simply reduce the large allocations to Equity and Commodity. The risk/return graph confirms the proximity of the optimal portfolios under the Sharpe, Adjusted Sharpe and Omega(0%) ratios, whereas the Omega(+3%) and Omega(+6%) optimal portfolios display much larger returns and volatilities, as one would expect.

In Exhibit 6, the results of the base case 'Equilibrium' scenario with IHF and FoHF proxies at their historical expected returns are opposite to the 'Historical' scenario in the sense that more than 100% of the allocations unrealistically go to FoHF and IHF proxies with Bond and Commodity at their –10% floor and Equity being slightly short as well, except for Omega(+6%). Again this should not surprise, given that Equity and the two hedge funds proxies have now the better expected returns and the hedge fund proxies have much lower volatilities than Equity. The IHF proxy is preferred to the FoHF proxy for the Sharpe, Adjusted Sharpe and Omega(0%) criteria because its volatility is lower than that of the FoHF proxy, but the FoHF proxy is preferred to the IHF proxy with Omega(+3%) and Omega(+5%).

The middle panel and the graph confirm the closeness of the optimal scenarios under the Sharpe, Adjusted Sharpe and Omega(+0%) ratios and indeed the small deviations from normality of the optimal portfolio return distributions. With no short sales allowed (bottom panel) all allocations go to the hedge funds proxies with again a marked preference for the IHF proxy over the FoHF proxy with Omega(0%) and the reverse with Omega(+3%). With Omega(+6%), the entire portfolio is in Equity.

But many investors might not agree with the base case 'Equilibrium' which attributes historical mean returns of 2.85% 4.86% to the IHF and FoHF proxies. To satisfy investors with different views about future hedge funds returns, we explore a range of possibilities with IHF expected returns ranging from 0% to +3% and FoHF expected returns from 0% to +5%. We display graphically the optimal allocations when maximising the Adjusted Sharpe Ratio in Exhibits 7i to 7vi. Remarkably, there are large allocations to either IHF or FoHF as soon as their expected returns exceed 1% and, correspondingly, allocations to Equity, Bond and Commodity fade away rapidly from their equilibrium levels to zero and even to the floor of –10%. Volatility, on the other hand, always has a small role to play. For all proxies except Volatility, optimal allocations are unstable; they change drastically for small changes in hedge fund expected returns. A summary of optimal portfolio structures for scenarios along the main diagonal of the previous graphs, that is for combined returns of FoHF and IHF from (5%, 3%) down to (2%, 0%) and then (1%, 0%) and (0%, 0%) is shown in Exhibit 7vii. From an essentially market equilibrium portfolio when the expected returns of hedge fund proxies are set at zero (right hand side), the FoHF proxy and then the IHF proxy quickly dominate as their expected returns are raised.

Turning now to the 'Subjective' scenarios a la Black-Litterman, and using the Adjusted Sharpe Ratio as our unique criterion, we observe the optimal allocations in Exhibit 8. To understand these results, it is important to observe how the expected returns vary from one scenario to another (recall that the expected return inputs for these scenarios are listed in Exhibit 4(iii)). For example, starting with the first scenario, the expected return of 0% for both IHF and FoHF is below their equilibrium returns of 0.54% and 0.69% respectively. Because Equity and, to a lesser extent, Commodity are positively correlated to the hedge fund proxies but Volatility is negatively correlated and Bond shows little correlation, the expected returns for Equity and Commodity are lower than at their equilibrium level but the expected return for Volatility is higher and that of Bond is about unchanged. Compared to equilibrium weights, this scenario produces a tilt from Commodity to Equity, Bond remains stable and hedge fund proxies do not appear. When the subjective forecasts (views) for hedge fund proxies increase, the posterior returns also increase, but not as fast; they are a weighted average between the equilibrium returns and the subjective views. The posterior return for Equity also increases and at a faster rate, carried by the positive correlations with the two hedge fund proxies and with more flexibility to change because of the higher volatility of Equity compared to the hedge fund proxies. The global net effect is that allocations to Equity and Bond decrease progressively as they are replaced first by FoHF and then by IHF. Commodity is shorted, when allowed, and there is a very small long Volatility position in all scenarios.

VII SUMMARY AND CONCLUSIONS

Investable Hedge Fund (IHF) indices have been designed to facilitate access to hedge funds for a wider class of investors. They are seen as an alternative to funds of hedge funds (FoHF) and their performance should be similar except for differences in selection criteria and trading costs. Other alternative investments such as commodities and volatility are also becoming more tradable and could also improve the performance of traditional passive equity and bond portfolios by providing either extra returns or better diversification. We have shown that a fair comparison between these alternative investments depends critically on expected return forecasts.

If we adopt a naïve historical view of returns, optimal allocations favour exclusively the few asset classes that happened to have performed well over the last few years, namely equities and commodities and exclude hedge funds. But investors, through their current allocations, show that they do not believe that past performance will persist. Rather, global market allocations, if rational, would indicate a completely different set of market equilibrium expected returns. Adopting this equilibrium view for all assets but hedge funds and varying forecasts for hedge fund expected returns from zero to their historical averages, one finds unstable allocations shifting rapidly from no allocations to hedge funds to unrealistically high allocations to hedge funds only.

A more reasonable comparison is achieved by attributing uncertainties to both equilibrium expected returns and subjective forecasts as if these were two independent and imperfect sources of information. We combine these two sources of information using the method first developed by Black and Litterman. We choose our uncertainty parameters to give about equal credibility to market equilibrium views and subjective forecasts. This choice is debatable, indeed we cannot find an intuitive interpretation for the uncertainty parameters in the BL framework, but by exploring a range of subjective views for hedge funds expected returns, we can still draw general conclusions. Both IHF and FoHF proxies should play an important role in enhancing passive bond/equity portfolios if we think that their expected excess returns are of the order of 1% or more. Our IHF proxy shows lower volatility than our FoHF proxy and might be preferred by conservative investors, but investors with a larger risk appetite should prefer the higher expected return of the FoHF proxy. Small, long volatility positions are also attractive, even when excess returns on these positions are expected to be negative (which is unrealistic over the long term). On the other hand, commodities should be shorted if one expects only small positive excess returns.

From a methodological perspective, the Black Litterman approach is the most reasonable among those we have tried here, but it has its drawbacks. The interpretation of market equilibrium views and subjective views as two independent sources of information is not realistic and confuses the assessment of uncertainties to each of these views. This may be the reason why BL has not become more popular. We intend to revisit this issue and suggest an alternative method where, on one hand, we assist a fund manager express her views and, on the other hand, we provide a natural path between the market views and the fund manager's personal views.

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Exhibit 1: Historical Performance of the Portfolio Constituents

Annual Excess Returns	Equity	Bond	Volatility	Commodity	IHF	FoHF
Mean	12.16%	1.27%	-18.18%	9.61%	2.85%	4.64%
Std. Dev.	10.45%	2.89%	34.61%	11.10%	3.73%	6.76%
Skewness	0.11	-0.09	0.26	-0.14	-0.04	-0.17
Excess Kurtosis	0.02	0.01	0.12	0.00	-0.06	-0.03
Jarque-Bera	0.10	0.06	0.53	0.15	0.02	0.23
1-mth autocorrelation	0.12	0.10	-0.32	-0.01	0.37	0.33
Sharpe Ratio	1.16	0.44	-0.53	0.87	0.76	0.69

Based on monthly arithmetic-returns for the period 1 October 2002 to 30 June 2006

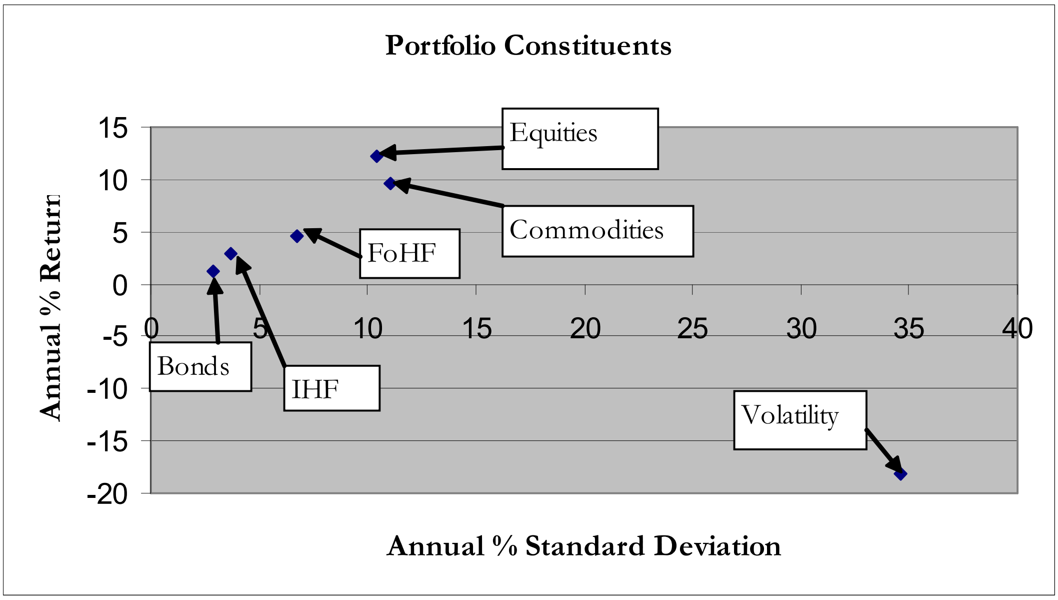


Exhibit 2: Correlation Matrix

	Equity	Bond	Volatility	Commodity	IHF	FoHF
Equity	1.00					
Bond	-0.15	1.00				
Volatility	-0.57	0.04	1.00			
Commodity	0.08	0.09	-0.15	1.00		
IHF	0.53	0.23	-0.41	0.50	1.00	
FoHF	0.47	0.20	-0.42	0.45	0.75	1.00

Based on monthly arithmetic-returns for the period 1 October 2002 to 30 June 2006

Exhibit 3: Market Equilibrium Weights as at 30 June 2006

	Equities*	Bonds**	Volatility	Commodities***	IHF +	FoHF ++
Market Values (USD bn)	40,460	62,767	0	16	20	426
Market Weights (%)	39.02%	60.53%	0.00%	0.02%	0.02%	0.41%

(*) Source: Standard and Poor's (**) Source: BIS data (Tables 13B and 16A)
(***) Source: Commitment of Traders (CoT) reports (+) Estimate based on announcements by the providers of the IHF (++) Source: Hedge Fund Research Inc, Industry Report 30 June 2006

Exhibit 4: Excess Return and Volatility inputs for the three scenarios

(i) Historical Returns Scenario

Historical Scenario	Equity	Bond	Volatility	Commodity	IHF	FoHF
Annual excess returns	12.16%	1.27%	-18.18%	9.61%	2.85%	4.64%
Annual volatility	10.45%	2.89%	34.61%	11.10%	3.73%	6.76%

(ii) Equilibrium Scenario

Equilibrium Scenario ($\gamma=4$)	Equity	Bond	Volatility	Commodity	IHF	FoHF
Annual excess returns	3.65%	0.09%	-8.14%	0.27%	2.85%	4.64%
Annual volatility	15.59%	2.80%	59.35%	10.32%	3.73%	6.76%

(iii) Subjective, Black-Litterman Scenarios

BL Posterior Return Inputs	Equity	Bond	Volatility	Commodity	IHF	FoHF
Views on Hedge Fund Returns						
Q1 - FoHF 5% : IHF 3%	7.24%	0.37%	-19.55%	2.57%	2.06%	3.57%
Q2 - FoHF 4% : IHF 2%	6.02%	0.27%	-15.85%	1.79%	1.53%	2.76%
Q3 - FoHF 3% : IHF 1%	4.81%	0.18%	-12.14%	1.02%	0.99%	1.94%
Q4 - FoHF 2% : IHF 0%	3.59%	0.08%	-8.44%	0.25%	0.45%	1.13%
Q6 - FoHF 1% : IHF 0%	3.22%	0.05%	-7.06%	0.01%	0.33%	0.71%
Q7 - FoHF 0% : IHF 0%	2.86%	0.02%	-5.68%	-0.22%	0.21%	0.29%

BL Parameters	Equity	Bond	Volatility	Commodity	IHF	FoHF
Equilibrium returns	3.64%	0.09%	-8.13%	0.27%	0.54%	0.69%
Posterior Annual Volatility	16.58%	3.01%	63.37%	10.98%	3.85%	6.98%

Tau (τ)	0.16

Exhibit 5: Historical Scenario – Summary of Allocations and Performance

Allocations	Equity	Bond	Volatility	Commodity	IHF	FoHF
Omega (0%)	37.94%	45.52%	2.70%	13.84%	0.00%	0.00%
Omega (3%)	71.46%	-10.00%	0.33%	38.21%	0.00%	0.00%
Omega (6%)	78.39%	-10.00%	-10.00%	41.61%	0.00%	0.00%
Adj. Sharpe	37.03%	43.30%	2.44%	17.23%	0.00%	0.00%
Sharpe	34.25%	46.07%	2.70%	16.97%	0.00%	0.00%

	PF. Ret	PF. Std	Skewness	Kurtosis	Sharpe	Adj. Sharpe	Omega
Omega (0%)	6.03%	4.05%	0.25	-0.03	1.49	2.00	73.33
Omega (3%)	12.17%	8.91%	0.15	-0.13	1.37	1.75	15.30
Omega (6%)	15.22%	12.00%	-0.07	-0.05	1.27	1.51	6.96
Adj. Sharpe	6.26%	4.18%	0.20	-0.08	1.50	2.01	1.17
Sharpe	5.89%	3.92%	0.18	-0.09	1.50	2.01	0.93

Allocations with positive weights constraints

Allocations	Equity	Bond	Volatility	Commodity	IHF	FoHF
Omega (0%)	35.21%	49.32%	2.66%	12.80%	0.00%	0.00%
Omega (3%)	65.00%	0.00%	0.00%	35.00%	0.00%	0.00%
Omega (6%)	71.38%	0.00%	0.00%	28.62%	0.00%	0.00%
Adj.Sharpe	34.32%	47.75%	2.29%	15.63%	0.00%	0.00%

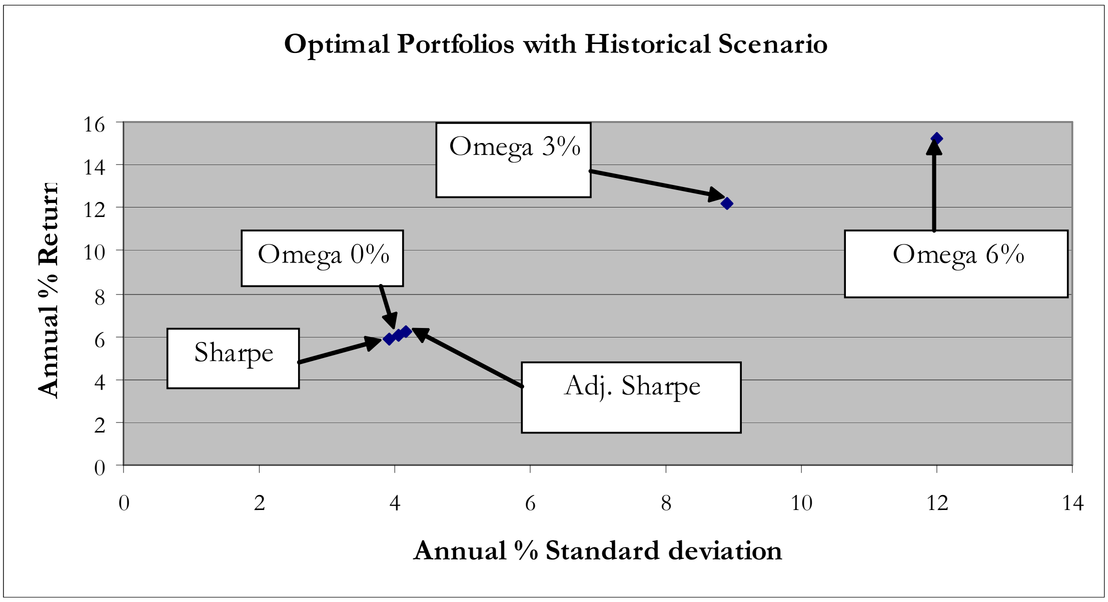


Exhibit 6: Equilibrium Base Case Scenario– Summary of Allocations and Performance

Allocations	Equity	Bond	Volatility	Commodity	IHF	FoHF
Omega (0%)	-6.97%	-10.00%	1.59%	-10.00%	100.00%	25.38%
Omega (3%)	-10.00%	-10.00%	-2.58%	-10.00%	32.58%	100.00%
Omega (6%)	30.00%	-10.00%	-10.00%	-10.00%	0.00%	100.00%
Adj. Sharpe	-7.09%	-10.00%	1.61%	-10.00%	100.00%	25.48%
Sharpe	-6.74%	-10.00%	1.58%	-10.00%	100.00%	25.16%

Performance	PF. Ret	PF. Std	Skewness	Kurtosis	Sharpe	Adj. Sharpe	Omega
Omega (0%)	3.61%	3.85%	0.18	0.06	0.94	1.06	11.61
Omega (3%)	5.38%	7.21%	0.01	-0.06	0.75	0.80	2.29
Omega (6%)	6.51%	13.60%	-0.34	0.20	0.48	0.48	1.10
Adj. Sharpe	3.61%	3.85%	0.18	0.06	0.94	1.06	0.21
Sharpe	3.61%	3.85%	0.17	0.06	0.94	1.06	0.21

Allocations based on positive weights constraints

Allocations	Equity	Bond	Volatility	Commodity	IHF	FoHF
Omega (0%)	0.00%	0.00%	1.85%	0.00%	76.90%	21.25%
Omega (3%)	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%
Omega (6%)	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Adj. Sharpe	0.00%	0.00%	1.88%	0.00%	76.47%	21.64%

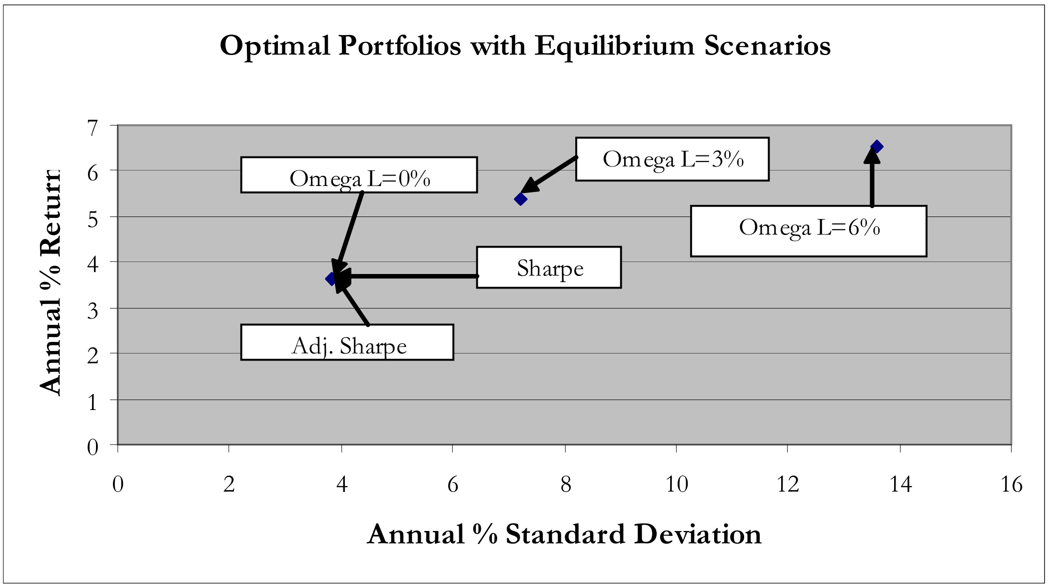


Exhibit 7: Equilibrium Scenarios – Optimal Allocations as a Function of Expected Returns for the Two Hedge Fund Constituents

Exhibit 7i: IHF

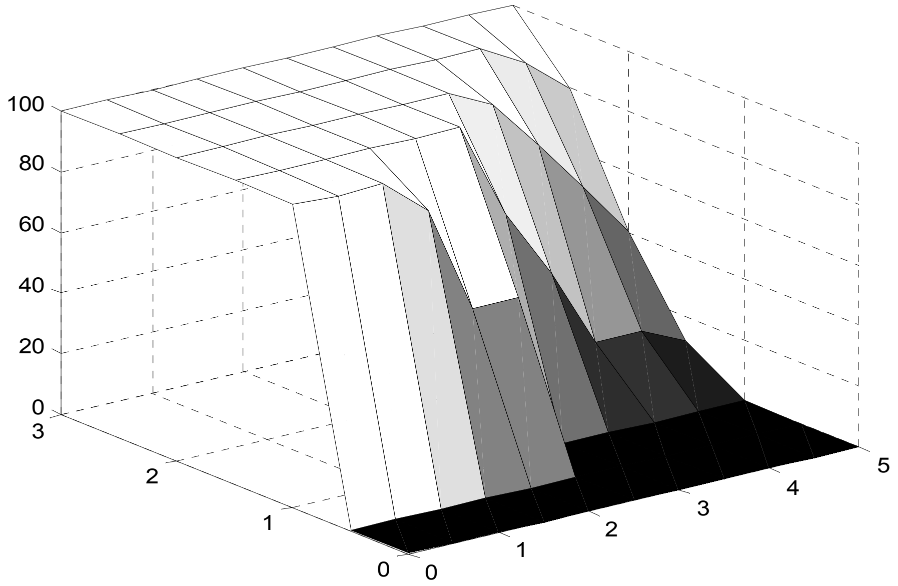


Exhibit 7ii: FoHF

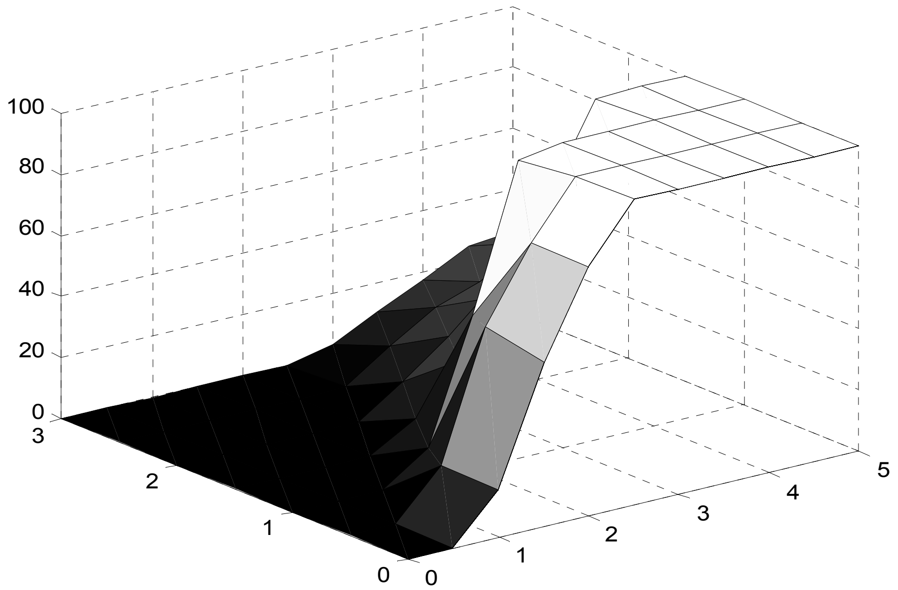


Exhibit 7iii: Equity

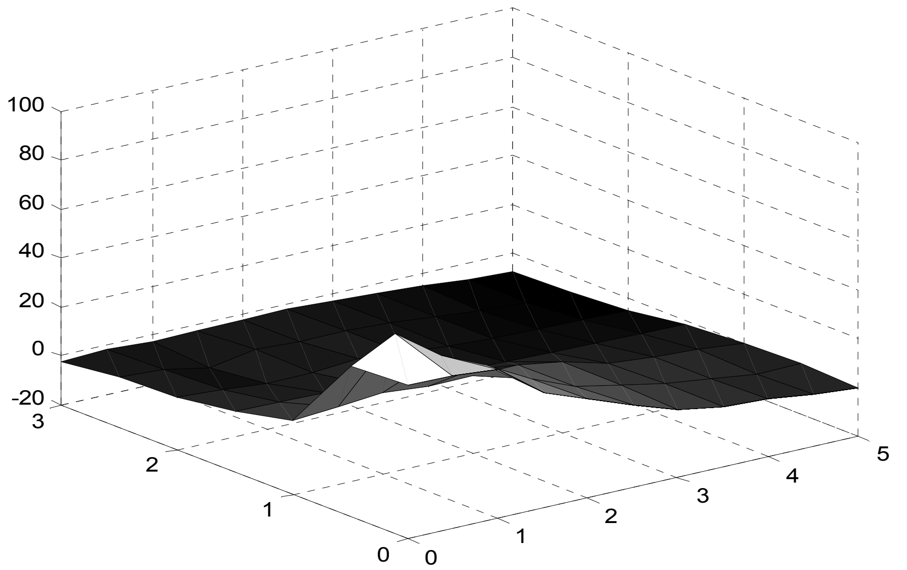


Exhibit 7iv: Bond

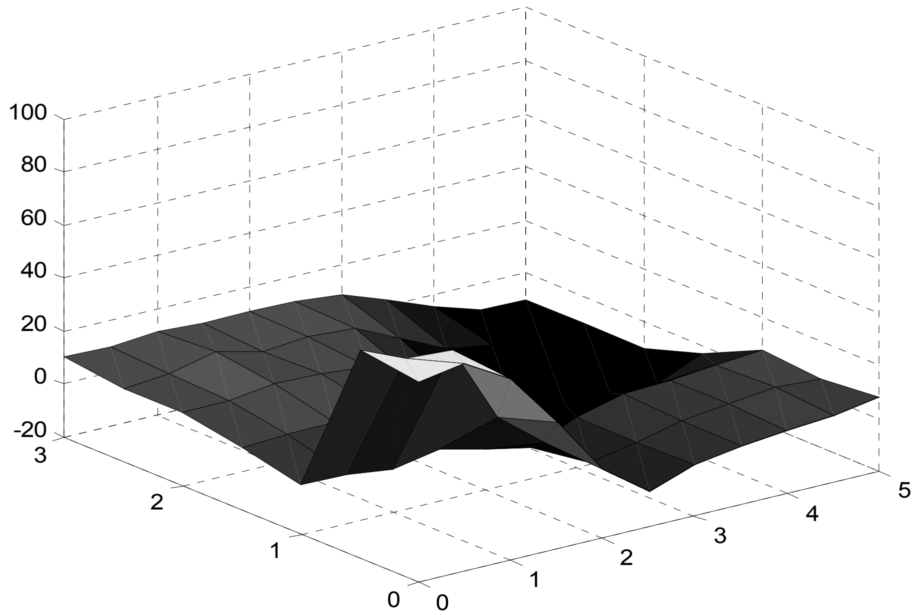


Exhibit 7v: Volatility

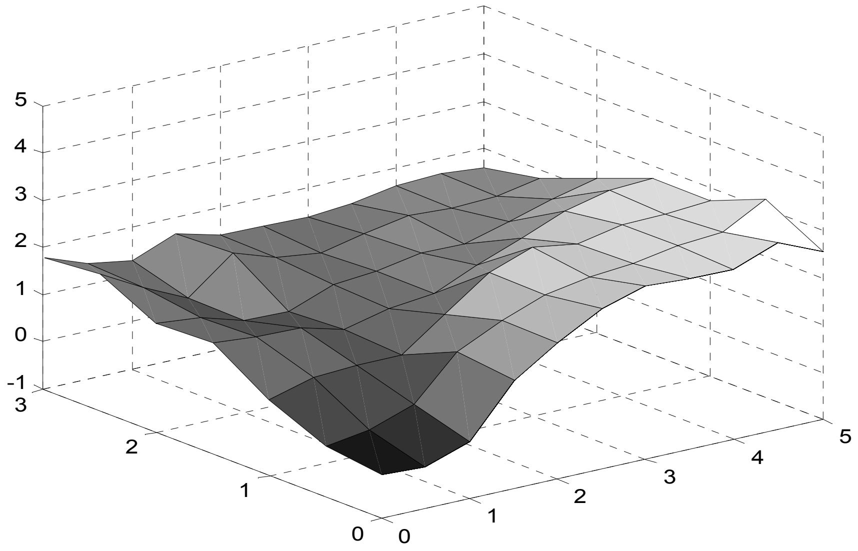


Exhibit 7vi: Commodity

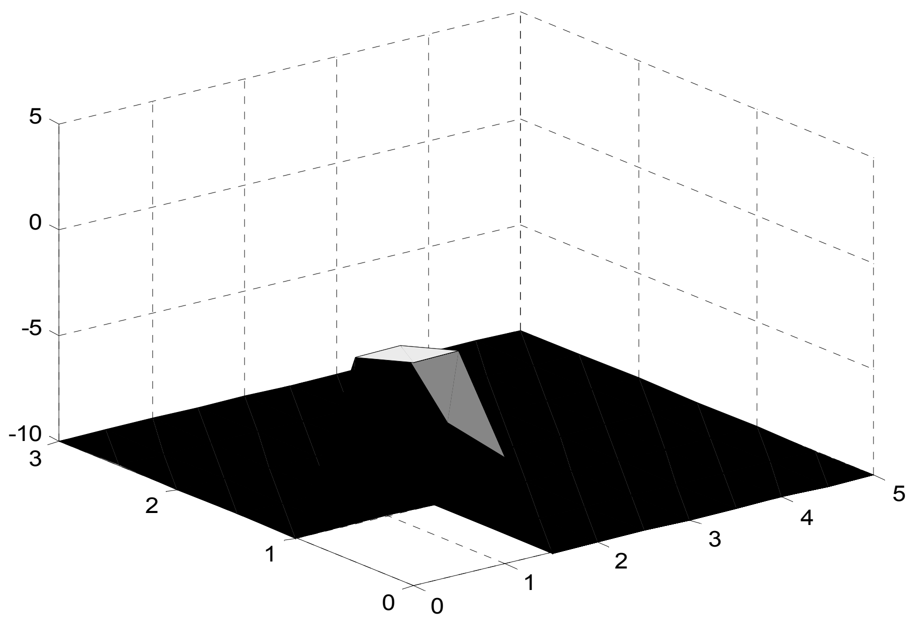
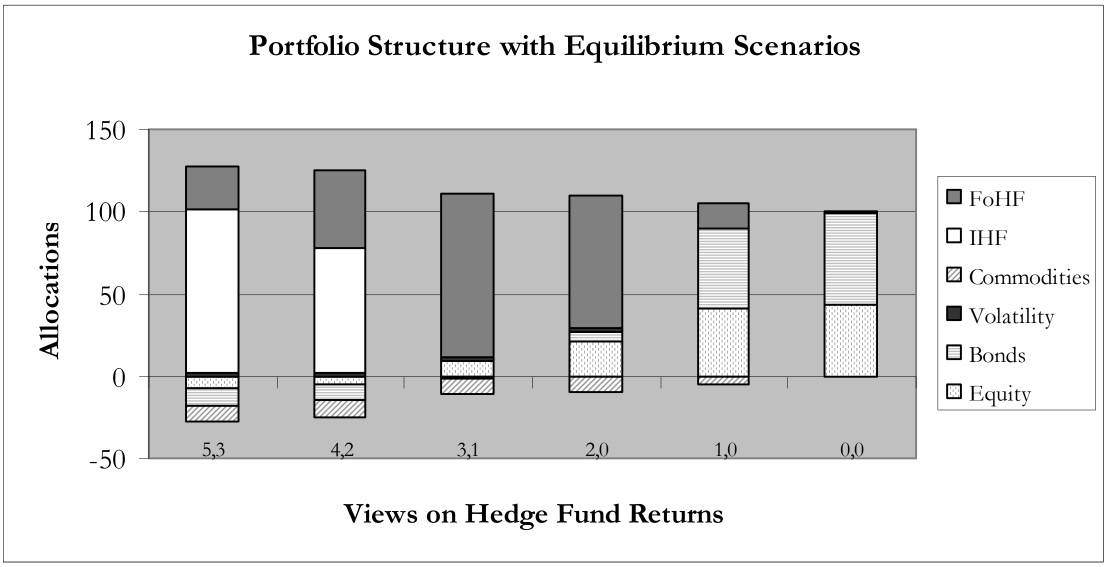


Exhibit 7vii: Diagonal Scenarios



Return Input Combinations	Equity	Bonds	Volatility	Commodities	IHF	FoHF
FoHF 5% : IHFI 3%	-7.82	-10.00	1.60	-10.00	100.00	26.2
FoHF 4% : IHFI 2%	-4.63	-10.00	1.79	-10.00	75.75	47.09
FoHF 3% : IHFI 1%	8.77	-1.35	2.57	-10.00	0.01	100.00
FoHF 2% : IHFI 0%	20.73	6.10	1.89	-9.99	0.00	81.28
FoHF 1% : IHFI 0%	40.89	48.53	0.21	-4.97	0.00	15.34
FoHF 0% : IHFI 0%	43.26	56.16	-0.05	0.63	0.00	0.00

Exhibit 8: Subjective Black-Litterman Scenarios – Summary of Allocations and Performance

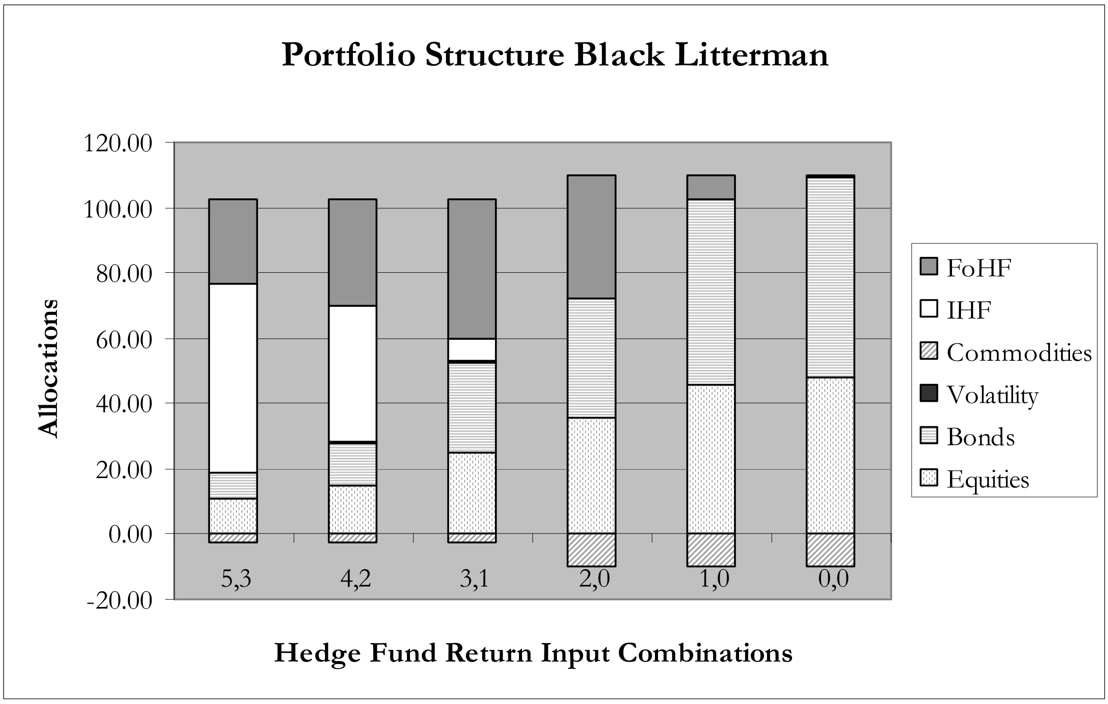
Allocations	Equity	Bond	Volatility	Commodity	IHF	FoHF
FoHF 5% : IHF 3%	10.69%	7.94%	0.16%	-2.40%	58.08%	25.53%
FoHF 4% : IHF 2%	14.85%	13.19%	0.14%	-2.43%	41.96%	32.29%
FoHF 3% : IHF 1%	25.23%	27.48%	0.30%	-2.79%	7.08%	42.70%
FoHF 2% : IHF 0%	35.47%	36.56%	0.30%	-10.00%	0.00%	37.67%
FoHF 1% : IHF 0%	45.94%	56.43%	0.11%	-10.00%	0.00%	7.52%
FoHF 0% : IHF 0%	48.17%	61.01%	0.82%	-10.00%	0.00%	0.00%

Performance	PF. Ret	PF. Std	Skewness	Excess Kurtosis	Sharpe	Adj. Sharpe	Omega
FoHF 5% : IHF 3%	2.82%	4.80%	0.11	-0.14	0.59	0.62	0.19
FoHF 4% : IHF 2%	2.39%	5.23%	0.12	-0.13	0.46	0.47	0.18
FoHF 3% : IHF 1%	2.10%	6.21%	0.17	-0.07	0.34	0.35	0.21
FoHF 2% : IHF 0%	1.68%	7.19%	0.24	0.06	0.23	0.24	0.23
FoHF 1% : IHF 0%	1.55%	7.73%	0.33	0.16	0.20	0.20	0.24
FoHF 0% : IHF 0%	1.37%	7.63%	0.37	0.21	0.18	0.18	0.23

Allocations based on positive weights constraints

Allocations	Equity	Bond	Volatility	Commodity	IHF	FoHF
FoHF 5% : IHF 3%	11.58%	8.29%	0.35%	0.00%	54.15%	25.63%
FoHF 4% : IHF 2%	15.78%	15.05%	0.32%	0.00%	38.62%	30.23%
FoHF 3% : IHF 1%	25.21%	28.76%	0.27%	0.00%	4.38%	41.38%
FoHF 2% : IHF 0%	35.09%	37.27%	0.23%	0.00%	0.00%	27.41%
FoHF 1% : IHF 0%	45.04%	54.53%	0.18%	0.00%	0.00%	0.24%
FoHF 0% : IHF 0%	49.69%	49.34%	0.98%	0.00%	0.00%	0.00%

Exhibit 9: Subjective Scenarios - Portfolio Structure for Various Hedge Fund Expected Returns



ENDNOTES

[1] Transparency refers to the index providers' ability to access information on the performance, risk exposures and operational systems of the hedge funds included in the indices.
[2] For example, in June 2006, the French FFR (Fonds de Réserves pour les Retraites) decided to allocate 10% to alternative investments but, contrary to many other pension funds (e.g. CalPERS), to exclude hedge funds.
[3] We shall refer loosely to these six portfolio constituents as asset classes although, strictly speaking, hedge funds are not an asset class but an investment vehicle and volatility is just an index on which tradable instruments are based.
[4] See for example the survey in early 2006 by Barclays Capital showing investors' intentions to increase their allocations to commodities [Kat & Oomen, 2006].
[5] Brazil, Russia, India and China.
[6] See www.federalreserve.gove 31 May 2006.
[7] Keeping some foreign exchange exposure on foreign investments would appear to generate a small positive expected return (Siegel's paradox) that improves the portfolio risk adjusted return as long as this exposure does not exceed 20% or so of foreign investments.
[8] Indeed, we have verified that we obtain similar conclusions when substituting the un-hedged MSCI equity index to the S&P500 index.
[9] The VIX, as first introduced in September 1993, was calculated from near the money S&P 100 index call and put options. The method of calculation was revised in September 2003; the new VIX is based on the prices of out-of-the money S&P 500 index call and put options using a special weighting scheme to mimic a log contract
[10] VIX futures daily trading volumes increased from less than 10,000 contracts per day in December 2005 to more than 50,000 per day in September 2006.
[11] The alternative GSCI index, for example, is heavily influenced by the energy sector. The current weights at May 2006 (per Reuters) are: energy 73%, agriculture 10%, industrial materials 10%, livestock 4% and precious metals 2%.
[12] We reviewed the offer documents for investment products based on the MSCI and HFR investable indices. Our assessment is also based on more general feedback from market professionals.
[13] With 45 observations, the standard error of the autocorrelation estimate when the true value is zero is 0.15.
[14] This filter has been used in different fields such as real estate [Geltner, 1993]. More general filters have been designed (see for example Getmansky et al., 2003) but do not seem necessary in this case
[15] We use lower case bold letters to represent vectors and upper case bold letters for matices.
[16] For example Bodie/Kane/Marcus in their widely used "Investments" textbook (McGraw Hill, 2005) use repeatedly a risk aversion coefficient of 4 in their illustrations. Litterman and He [1999] use a value of 2.5
[17] From Black &litterman [1992] p.34. Most commentators have also argued that the proportionality constant, τ, should be much less than 1, but our argument shows that it is not necessarily the case.
[18] Actually, there are now some investment vehicles that allow investors to short some investable indices.

